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(Short Communication)

Methodological notes.
Consistent definition of the local equivalence principle

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Abstract

The opinion that the mathematical training of physicists, even leading ones, is insufficient is confirmed. These are the claims of V. A. Fock and N. N. Bogolyubov was presented to Landau. Mathematics is not reduced to a set of formulas and solutions to equations. A new formulation of the local principle of equivalence of the gravitational field and the accelerated frame of reference based on the standard mathematical epsilon-delta method is proposed.

Keywords: frame of reference, equivalence principle, epsilon-delta method, tangent space.

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1. Introduction

1.1 Minkowski

Minkowski formulates the concept of physical space as a four-dimensional metric space-time [1]. However, the time coordinate in this space is no longer independent of the other coordinates. The course of the clock depends on the speed and on the gravitational potential:

> "*Thanks to the theory of relativity, the possibility of a four-dimensional interpretation of the "world" appears, since in this theory time loses its independence*...", Einstein [2].

Already in the special theory of relativity, space appears as an independent physical object. This theory considers only the properties of space-time that determine the movement of bodies.

Einstein introduced the concept of a relativistic frame of reference, considering the frame of reference as a set of rulers and clocks. Further development of the "theory of space" led to Einstein's relativistic theory of gravity. Here the physical space loses its independence, the space itself (more precisely, its fundamental tensor) is set by material bodies, including the electromagnetic field. The gravitational field equation defines the metric tensor.

In 1907, Einstein considers the equivalence principle [3] as a means to transfer the results obtained for accelerated reference systems to systems with a gravitational field. At the same time, Einstein limited himself to systems with a homogeneous field, which of course is not suitable for arbitrary reference systems. The modern approach to the equivalence principle is distinguished by a variety of interpretations and definitions, for example, [4], [5] and [6].

Talking about the local equivalence of the frames of reference of the Riemann space and the Minkowski space.

1.2 Landau and Lifshitz

In the cult book [4], Landau and Lifshitz try to prove the local equivalence of a system with a gravitational field to an inertial system:

> "*Formula (85.15) under condition (85.16) allows us to prove the above statement about the possibility of such a choice coordinate system for which all Γ^i_{kl} vanish at any predetermined point (such a system is called locally inertial or locally geodesic, see §87). Indeed, let a given point be chosen as origins and quantities Γ^i_{kl} have in it initially (in x^i coordinates) values $\left(\Gamma^i_{kl}\right)_0$. We will produce near this point transformation*
>
> $$x'^i = x^i + \frac{1}{2}\left(\Gamma^i_{kl}\right)_0 x^k x^l \qquad (85.18)$$
>
> *Then*
>
> $$\left(\frac{\partial^2 x'^m}{\partial x^k \partial x^l}\frac{\partial x^i}{\partial x'^m}\right)_0 = \left(\Gamma^i_{kl}\right)_0 \qquad (85.19)$$
>
> *and according to (85.15)[2] all Γ^i_{kl} vanish.*"

Here see that the authors consider the transition from the first expression to the second by incorrectly replacing small values of the x^i coordinates with differentials, but

$$x^i = \partial x^i + o(X).$$

The rejection of the finite value $o(X)$, contrary to popular belief, cannot be perceived as a coordinate transformation. Replacing coordinates with their differentials means replacing geodesics with their tangents at the point P.

1.3 Meller's

A more transparent approach is used in Meller's excellent monograph [7] (§ 9.6. Local pseudo-Cartesian coordinates and local inertial systems). Meller comes from the curvilinear coordinates X to the local (near the origin) pseudo-Euclidean system $\check{X}$ using the transformation

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[2] This is the coordinate transformation formula $\Gamma^i_{kl} = \Gamma'^m_{np}\frac{\partial x^i}{\partial x'^m}\frac{\partial x'^n}{\partial x^k}\frac{\partial x'^p}{\partial x^l} + \frac{\partial^2 x'^m}{\partial x^k \partial x^l}\frac{\partial x^i}{\partial x'^m}$.

$$\breve{x}^i = \frac{\partial \breve{x}^i}{\partial x^k} x^k; \tag{2}$$

$$x^i = \frac{\partial x^i}{\partial \breve{x}^k} \breve{x}^k,$$

here the derivatives are calculated at the origin. We note right away that the inverse transformation (3) is impossible, since transformation (3), as well as transformation (2), is a transformation into the gallium space. After such a transformation, all information about the parameters of the original Riemann space is lost, which cannot be restored.

Let us show that in this case the equal sign in (2) is inappropriate. Indeed, the equalities must be written in terms of the total differential

$$\breve{x}^i = d\breve{x}^i + o(X) \approx d\breve{x}^i; \tag{4}$$

And thus, we can only speak of an asymptotic transition from curvilinear coordinates to pseudo-Euclidean ones

$$\breve{x}^i \to d\breve{x}^i.$$

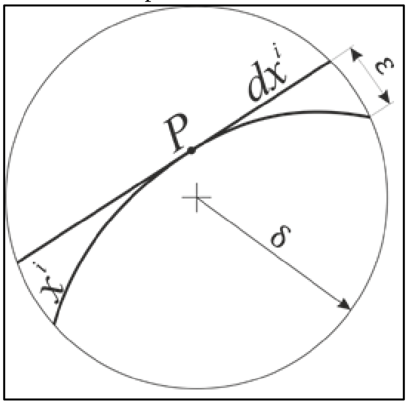
As the radius δ of the neighborhood decreases (Fig. 1), the geodesics begin to look more and more like straight lines. In other words, a small neighborhood of the pseudo-Riemannian space can be arbitrarily close to the inertial frame (Einstein's freely falling elevator), but never coincides with it. In this case, as it should be in a finite region, neither the metric tensor nor, depending on it, the Riemann tensor with its convolutions and Christoffel symbols.

2. Illustrate this

In curvilinear coordinates, the geodesic line will never coincide with the tangent in the vicinity of the tangent point, no matter how small this neighborhood is. But this geodesic in the zero limit of the neighborhood will coincide with its own tangent, and the equation of motion is simplified to the equation of motion in the inertial frame

$$d^2x^i/ds^2 = 0.$$

Figure 1: Transformation (4) is a transition from the original space with coordinates x^i to the inertial tangent space with a common point P.



Source: [1-7]

3. Conclusions

Formulate the principle of space equivalence to the point P to the pseudo-Euclidean space.

If the point P belongs to an open neighborhood Q of the space X and has a derivative at the point P, then this space is locally equivalent to the tangent space $\breve{X}$ at the point P if, for any positive ε, there is a neighborhood radius δ such that for any $x^i \in Q$ will be carried out $\varepsilon > |x^i - dx^i|$.

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